Investigating the effects of temperature non-uniformity on supersonic jet noise with large-eddy simulation

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Large eddy simulations are performed for heated over-expanded supersonic jets issued from a military-style faceted converging-diverging nozzle. To investigate the impact of inlet temperature non-uniformity on the radiated noise, three different flow temperature conditions are examined: military power settings, afterburning conditions and non-uniform inlet temperature, consisting of an annulus of afterburned exhaust and a central stream of military power exhaust. The comparisons with the available measurements from NASA at military power conditions show good agreement, in particular when the experimental core-bypass flow upstream of the nozzle is modeled: the LES flow field and shock structures closely match the PIV data and the far-field noise predictions are within 1-2 dB of the microphone data for most relevant angles and frequencies. The preliminary results from the simulation of the non-uniform inlet temperature conditions indicate a reduction in the noise output of the jet compared to the afterburning conditions. This noise mitigation is observed over a large frequency range and for most inlet angles, with decrease in overall peak levels by 1.6 dB and radiated power by 1.1dB, independent of performance losses.

Nomenclature

\[ BPR \quad \text{Mass bypass ratio} \]
\[ c \quad \text{Speed of sound} \]
\[ D \quad \text{Nozzle exit diameter} \]
\[ dt \quad \text{Time step} \]
\[ f \quad \text{Frequency} \]
\[ M \quad \text{Mach number} \]
\[ \dot{m} \quad \text{Mass flow rate} \]
\[ NPR \quad \text{Nozzle pressure ratio} \]
\[ NTR \quad \text{Nozzle temperature ratio} \]
\[ p \quad \text{Pressure} \]
\[ Re \quad \text{Reynolds number} \]
\[ St \quad \text{Strouhal number} \]
\[ T \quad \text{Temperature} \]
\[ t \quad \text{Time} \]
\[ t_{sim} \quad \text{Total simulation time} \]
\[ U_j \quad \text{Mean streamwise jet velocity} \]
\[ u, v, w \quad \text{Fluid velocity components} \]
\[ x, y, z \quad \text{Cartesian coordinates} \]
\[ \mu \quad \text{Dynamic viscosity} \]
\[ \phi \quad \text{Jet inlet angle} \]
\[ \rho \quad \text{Density} \]

\text{Subscript}

\[ \infty \quad \text{Free-stream property} \]
\[ b \quad \text{Bypass stream property} \]
\[ \bar{b} \quad \text{Core stream property} \]
\[ j \quad \text{Fully-expanded jet conditions} \]
\[ t \quad \text{Total (stagnation) property} \]

\text{Superscript}

\[ ' \quad \text{Disturbance quantity} \]
\[ \bar{\quad} \quad \text{Time average} \]

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I. Introduction

As military jet engine technology advances, noise reduction of the exhaust jet remains a persistent challenge for supersonic engine designs. While subsonic commercial engines have exploited higher bypass ratios and chevrons to significantly reduce noise, similar improvements have yet to be seen in military applications. In tactical jets, supersonic exhaust speeds and high temperature ratios drive excessive noise, posing safety risks to personnel working around the aircraft, and general annoyance to the public in the vicinity of airbases. Detailed measurements in full-scale engines are costly and difficult, and most lab facilities are limited to smaller scale jets at lower temperatures. With advancements in high-performance computing and numerical methods, large eddy simulations (LES) can arguably provide insight on high-speed heated jets and their acoustic field in a more flexible and cost-effective way than most in situ or lab testing, with the potential to drive improvements on performance, durability, and noise mitigation.

Recent experimental efforts\textsuperscript{1–3} have investigated new noise reduction concepts in supersonic military-style nozzles by focusing on the effects of exhaust jet temperature and temperature distortion profiles. In these experiments, temperature non-uniformities are produced through auxiliary nozzle hardware that introduces a stream of colder fluid inside the exhaust system, upstream of the nozzle exit. Similar temperature non-uniformity could potentially be achieved in high-performance exhaust systems if, for instance, only the outer rings of the afterburner are activated (see figure 1). This would generated a “cooler” temperature stream of initial length scale $L$ centered in the core of the jet. Such a configuration also results in complex mixing between streams of various combustion products and ambient air, and gives rise to non-uniformities in the thermophysical properties of the exhaust gas. In addition, the increased thermal variability can cause larger fluctuations in the unsteady heat loads to nozzle hardware. Unfortunately, these scenarios cannot be reproduced in most lab facilities that lack the engine core and fuel systems to produce realistic high temperature ratios and combustion streams.

![Figure 1. Afterburner configuration of a MiG-23, with the aft end of the engine centerbody in the middle and three rings of flameholders. The length scale $L$ is representative of the inner ring diameter.](image)

In this study, large eddy simulations are performed to investigate the impact of inlet temperature non-uniformity on the noise generated by hot over-expanded supersonic jets. The highly heated flow is issued from a military-style faceted converging-diverging nozzle and three different flow temperature conditions are examined: military power settings, afterburning conditions and non-uniform inlet temperature. As a first step, the present work focuses solely on temperature effects and for now, the presence of the combustion products are ignored in the simulations. The nozzle configuration, operating conditions and numerical setup are described in section II. The initial blind comparisons with available measurements at military power conditions are discussed in section III, along with the results of subsequent simulations with increased grid resolutions and improved modeling of the experimental core-bypass flow upstream of the nozzle. For the three flow temperature conditions, the preliminary analysis is presented in section IV, including far-field noise predictions and spectral proper orthogonal decomposition (SPOD).
II. Jet configuration and numerical setup

A. Nozzle geometry and experimental setup

This investigation focuses on supersonic flow through a GE F404-like nozzle that has previously been studied experimentally at NASA\cite{4,5} and at The Pennsylvania State University.\cite{6,7} The geometry is a sharp-throat converging-diverging faceted nozzle, with 12 large facets, and 12 smaller facets connecting the larger ones. The nozzle diameter is approximately $D \approx 5$ inches and the design Mach number is $M_d = 1.65$. Additional details on the nozzle along with its dimensions can be found in figure 2.

![Nozzle schematic and dimensions](image)

Figure 2. Nozzle schematic and dimensions

The experiments were done at the Nozzle Acoustic Test Rig (NATR) in the AeroAcoustic Propulsion Laboratory (AAPL) at NASA Glenn Research Center. The acoustic measurements by Henderson and Bridges\cite{5} were made with the far-field array of microphones presented in figure 3(a), on a 45 foot constant radius arc, from inlet angle $\phi = 45^\circ$ to $160^\circ$, every $5^\circ$. The data is corrected for atmospheric absorption and provided on a one-foot lossless arc. The spectra are expressed as normalized Power Spectral Density ($PSD$) in dB/Hz (relative to $20 \times 10^6$ Pa). The nozzle was supplied with air streams at realistic pressures and temperatures using the dual-stream High Flow Jet Exit Rig (HFJER), which includes an internal axisymmetric splitter. Because of the high temperature of the core stream, the rig could not be operated for long without cold air flowing in the bypass duct to cool the nozzle, as shown schematically by the cooling film flow in figure 3(b). Similarly, in tactical aircraft with afterburner exhausts, bleed holes are typically used to introduced cooler bypass flow into the nozzle. As reported by Bridges, Wernet & Frate,\cite{4} the bypass-to-core area ratio is 0.2 in the experiment, leading to a mass bypass ratio of approximately $BPR = 0.3$, which is close to the published value of the F404 engines. The experimental conditions are summarized in table 1, including the mass flow rates $\dot{m}_b$ and $\dot{m}_c$ for the bypass and core streams, non-dimensionalized by $\rho_\infty c_\infty D^2$.

![Details of the experimental configuration](image)

(a) Photograph of the AAPL (from Ref. 5)  (b) Schematic of the core-bypass splitter (from Ref. 4)

Figure 3. Details of the experimental configuration
Table 1. Summary of experimental conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>Nozzle</th>
<th>Setpoint</th>
<th>(\text{NPR}_c)</th>
<th>(\text{NPR}_b)</th>
<th>(\text{NTR}_c)</th>
<th>(\text{NTR}_b)</th>
<th>(\dot{m}_c)</th>
<th>(\dot{m}_b)</th>
<th>(M_\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Military Power (BPR 0.3)</td>
<td>G6S1</td>
<td>44540</td>
<td>3.51</td>
<td>3.51</td>
<td>2.99</td>
<td>1.05</td>
<td>0.543</td>
<td>0.157</td>
<td>0.01</td>
</tr>
</tbody>
</table>

B. LES Methodology

The large-eddy simulations were performed using the unstructured compressible flow solver “Charles”\(^8\) developed at Cascade Technologies, Inc. The present version of the solver uses a novel mesh generation paradigm based on the computation of Voronoi diagrams,\(^9\) from the specification of the relevant surface geometry and the set of points (i.e., generating points) where the solution is to be sampled. A Voronoi diagram uniquely divides the volume in the computational domain based on Euclidean distance, from the prescription of the geometry and point cloud alone: connectivity and topology are unique and deterministic consequences of the points and surface, as shown in figure 4. This approach drastically simplifies the meshing of complex geometry and results in high-quality, nearly-isotropic grids, well-suited for large-eddy simulations and aeroacoustics predictions.

![Generating points only](image1)

![Generating points and boundary surface](image2)

**Figure 4.** An arbitrary set of generating points (black circles) and the associated 2D Voronoi diagram (black lines). On the right a boundary surface (red lines) confines the mesh to a specific volume

As in previous work with the Charles solver, the nondimensionalization is based on the nozzle diameter \(D\) and the ambient speed of sound \(c_\infty = \sqrt{\gamma p_\infty / \rho_\infty}\). The resulting form of the ideal gas law is \(p = \rho T / \gamma\), with constant specific heat ratio \(\gamma = 1.4\). For now, combustion products and multi-species thermodynamic effects are ignored. The nozzle pressure ratio and nozzle temperature ratio are defined as \(\text{NPR} = P_t / P_\infty\) and \(\text{NTR} = T_t / T_\infty\), where the subscript \(t\) and \(\infty\) refer to the stagnation (total) property and free-stream conditions, respectively. For the present over-expanded conditions, the jet Mach number and acoustic Mach number are defined as \(M_j = U_j / c_j\) and \(M_a = U_j / c_\infty\), where \(U_j\) is the mean (time-averaged) streamwise jet velocity and the subscript \(j\) refer to the (equivalent) fully-expanded jet properties. The Reynolds number is \(Re_j = \rho_j U_j D / \mu_j\) and the experimental value is matched in the simulations.

The nozzle geometry (with exit centered at \((0, 0, 0)\)) is explicitly included in the computational domain, which extends from approximately \(-3D\) to \(60D\) in the streamwise (x) direction and flares in the radial direction from \(20D\) to \(40D\). The center plug and the axisymmetric splitter upstream of the nozzle were not initially provided and therefore, these geometries are not included in the current simulations. A very slow coflow at Mach number \(M_\infty = 0.01\) is imposed outside the nozzle, matching the experimental conditions. Sponge layers and damping functions are applied to avoid spurious reflections at the boundary of the computational domain.\(^{10, 11}\) The Vreman\(^{12}\) sub-grid model is used to account for the physical effects of the unresolved turbulence on the resolved flow. To try to capture the internal boundary layer and nozzle-exit turbulent state, which has been shown to be important for flow field and noise predictions,\(^{13}\) wall-stress modeling based on the equilibrium boundary layer assumption\(^{14-16}\) and near-wall grid refinement are used for the interior surface of the nozzle, in the convergent, throat, and divergent sections.
The far-field noise is computed using the frequency-domain permeable formulation\(^{17}\) of the Ffowcs Williams \& Hawkings\(^{18}\) (FW-H) equations. The transient data is recorded on a conical FW-H surfaces extending to \(x = 30D\). For the treatment of the FW-H outflow disk, the method of “end-caps” of Shur et al.\(^{19}\) is applied for \(x > 20D\), where the complex far-field pressure predicted from eleven FW-H surfaces with the same shape but outflow disks at different streamwise locations are phase-averaged. The noise predictions are done at the location of the experimental microphone array on a 45-foot arc, assuming lossless propagation. For each microphone location, the FW-H calculation is performed at 36 stations equally-spaced in the azimuthal direction, and the narrow-band spectra are azimuthal-averaged. For all the experimental and numerical data, a bin-average power spectral density (PSD) is computed, with bin size \(df = 250\) Hz.

C. Operating conditions

In the initial study done during the Center for Turbulence Research (CTR) Summer Program 2018 at Stanford University,\(^{20}\) three different engine operating condition were considered, all without bypass flow. The first condition simulated is a flow at military power settings, for comparisons with the experimental data provided by NASA.\(^4,5\) The second case is a flow at an afterburning condition, to characterizes the noise increase with regard to temperature. Finally, the third condition corresponds to a flow with non-uniform temperature profile, consisting of an annulus of afterburned exhaust and a central stream of military power exhaust, which is compared to the afterburning condition to assess potential noise reduction. The spatially varying boundary condition for the non-uniform case was achieved by prescribing a hyperbolic tangent profile radially at the nozzle inlet, as shown in figure 5. Based on existing afterburner design like the one shown in figure 1, a single inner “cooler” spot is placed on the centerline and its estimated length scale is chosen as \(L \approx 0.5D\). The initial mesh used for these cases is about 57 million nearly-isotropic cells, with finest resolution of \(\Delta/D = 0.005\) along the walls and within the shear layer of the jet, and \(\Delta/D = 0.01\) in the jet plume up to \(x/D = 15\).

This initial work was performed without any prior knowledge of the experimental data. Based on the results and the blind comparisons with the NASA measurements at military power conditions, an additional simulation was subsequently performed. First, the mesh resolution in the near-nozzle exit region was slightly increased to better capture the first shock cells. Second, a dual plug-flow profile was imposed at the nozzle inlet, for both streamwise velocity and temperature, to more closely match the NASA configuration with bypass ratio \(BPR = 0.3\) (see figure 5). These profiles were computed using the isentropic flow equations to match the experimental mass flow rates, NPR and NTR for both core and bypass streams. The cooler, slower bypass flow originally extends approximately 0.05D off the walls and the resulting bypass-to-core area ratio is 0.17, slightly lower than the experimental value because the center plug is not accounted for. The mesh resolution near all the interior surfaces of the nozzle and along the lipline was also increased such that...
the internal cooler flow and the shear-layer between the bypass and core streams were completely enclosed in the region of finest resolution, from the inlet up to 1D downstream of the nozzle exit. These changes lead to an increase in grid size to about 66 million nearly-isotropic cells.

Table 2 provides a breakdown of the operation conditions for all the cases, along with details of the computations, including mesh size and total simulation time $t_{\text{sim}}$ for the collection of data and statistics, after the initial transient is removed. All cases have the same nozzle pressure ratio as it was assumed the added combustion occurs at constant pressure. In addition, there are no changes in nozzle geometry, even for the afterburner conditions. These assumptions, along with the ideal gas equation of state at constant $\gamma$, lead to each case operating at a condition of equivalent thrust. While this is not a completely accurate representation of on-wing nozzle behavior, it allows for a comparison of noise levels independent of influence from thrust reductions. For the cases with varying inlet profiles, the definitions of the acoustic Mach number, $T_j$ and Reynolds number number can be ambiguous since the values are different for the core and outer streams. Strictly speaking, the fully-expanded values of an equivalent single stream jet should be computed. In literature, this equivalent basis is often useful, but it is not perfect. For now, as a simplifying convention, we chose to present the LES data non-dimensionlized by the properties of the stream with the highest mass flow rate, and the corresponding values are listed in Table 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mesh size (10^6 cells)</th>
<th>splitter BPR</th>
<th>NPR core</th>
<th>NPR outer</th>
<th>$M_j$</th>
<th>$M_a$</th>
<th>$T_j/T_\infty$</th>
<th>$Re_j$</th>
<th>$t_{\text{sim}}c_\infty/D$</th>
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<tbody>
<tr>
<td>Military Power</td>
<td>57</td>
<td>0</td>
<td>3.51</td>
<td>2.99</td>
<td>2.99</td>
<td>1.47</td>
<td>2.13</td>
<td>2.09</td>
<td>2E6</td>
</tr>
<tr>
<td>Afterburner</td>
<td>57</td>
<td>0</td>
<td>3.51</td>
<td>7</td>
<td>7</td>
<td>1.47</td>
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<td>Non-uniform</td>
<td>57</td>
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<td>3.51</td>
<td>2.99</td>
<td>7</td>
<td>1.47</td>
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<td>4.89</td>
<td>0.7E6</td>
</tr>
<tr>
<td>Military Power</td>
<td>66</td>
<td>0.3</td>
<td>3.51</td>
<td>2.99</td>
<td>1.05</td>
<td>1.47</td>
<td>2.13</td>
<td>2.09</td>
<td>2E6</td>
</tr>
</tbody>
</table>

Table 2. Summary of operating conditions and simulation parameters.

III. Comparisons with experiments at military power conditions

A. Flow field

Figure 6 show the instantaneous temperature and the velocity magnitude for the cases at military power conditions with and without the modeled bypass stream. Since the jet is over-expanded, a complex but relatively steady shock train is formed and the jet contracts after the first shock. Both simulations feature similar shock structure in the jet plume because the nozzle pressure ratio is unchanged. The total mass flow rate through the nozzle was monitored during the simulations, and $\dot{m}_{\text{total}} \approx 0.7$ for both cases, matching the experimental value. The main difference is the presence of a thin near-wall film of cooler, slower fluid introduced at the inlet. Here, the dual plug-flow profile was chosen as a simple way to model the experimental bypass flow. However, it introduces large velocity and density gradients at the interface between the two streams, which quickly lead to Kelvin-Helmholtz instability. Within 0.5D of the inlet, the internal shear-layer starts to roll-up and transitions to turbulence with large vortical structures shedding off the walls. These features are likely enhanced by the specific choice of inlet profiles and probably less coherent (if present at all) in the experiment with the center plug and the axisymmetric splitter. No attempts were made to optimize the profiles, as the next logical step would be to directly include the experimental core-bypass geometries in the computational domain. With the Voronoi-based grid generation capabilities, this presents no meshing challenges and would lead to only a modest increase in mesh size and computation cost. Nevertheless, thanks to the strong favorable pressure gradient throughout the nozzle, these vortical structures are washed out as the flow accelerate pass the sharp throat and the dual plug-flow profile ultimately results in a much thicker, turbulent boundary layer on the diverging section of the nozzle and thicker shear layer.

The impact of the bypass flow on the nozzle exit profiles and early-jet plume development is also clearly visible in the time-averaged and rms contours of the streamwise velocity presented in figure 7, in the midsection plane through a large facet. As described by Bridges, Wernet & Frate, the PIV measurements close to the nozzle exhibit a mean velocity deficit which is a legacy of the small annulus of cooler, low-speed bypass flow upstream of the nozzle throat. The RMS contours are also broader, extending below the lipline, with a
Figure 6. Instantaneous temperature $T/T_\infty$ (top image) from 1 (black) to 2.5 (white) and velocity magnitude $|u|/c_\infty$ (bottom image) from 0.1 (blue) to 2.3 (red) for the military power conditions with and without the bypass flow.
double peak near the nozzle exit because of the second inner shear layer with the hotter, higher-speed core flow. These features are now better captured in the simulation with the dual plug-flow profile at $BPR = 0.3$. In contrast, the single stream case has a much thinner nozzle-exit boundary layer and shear layer, with a single peak and higher values in the turbulence levels along the lipline. Aside from that slight reduction in the peak RMS levels in the shear layer, the differences between the two configurations are mainly limited to the near-nozzle region up to $x/D = 1$. For both cases, the LES predictions of the complex shock structure closely match the PIV measurements in terms of location and strength.

![Comparison of PIV and LES](image)

Figure 7. Comparisons between PIV and LES for the time average (a) and rms (b) of streamwise velocity at the military power conditions with and without the bypass flow.

B. Radiated noise

The impact of the bypass flow on the radiated noise is less intuitive and more challenging to anticipate a priori. There are no acoustics measurements for the heated jet on this nozzle configuration at $BPR = 0$ because the jet rig could not operate long without the bypass cooling. On one hand, Bridges, Wernet & Frate suggested that the second shear-layer could provide additional noise sources not present in the single-stream configuration. On the other hand, the velocity deficit introduced by the low-speed flow near the lipline could reduce convective velocities and therefore the Mach wave radiation. The lower peak turbulence levels along the shear layer could also translate into lower fine scale noise radiation. Furthermore, as the bypass stream mostly affect the flow for $x/D < 1$ and leaves the extended shock train mostly unchanged, the broadband shock associated noise (BBSAN) might not be altered. At the same time, the thicker mean profile in that near-nozzle region could change some of the grow rates of the instability waves and potentially lead to different characteristics for the wavepackets.

To begin to shed some light into these competing issues, the LES noise predictions with and without the bypass flow are compared to the NASA microphone measurements in figure 8. Given that the simulation at $BPR = 0$ was performed without any prior knowledge of the experimental data, the initial blind comparison was encouraging. At the upstream and side angles, there seems to be a secondary peak around $f = 1250$ Hz and additional low-frequency noise sources in the experiments that are not present or captured in neither cases. The peak noise at $\phi = 130^\circ$ is also slightly overestimated in the simulations. Nevertheless, these
Figure 8. Noise spectra and OASPL directivity at military power conditions with and without the bypass flow: $BPR = 0$ (LES ---); $BPR = 0.3$ (Experiment o ; LES ---)
predictions captured the correct spectra trends and OASPL directivity, with levels within 2-3 dB for most relevant angles and frequencies.

The predictions for the $BPR = 0.3$ case show some improvements in the spectra shapes and levels, mostly for the higher frequencies and the BBSAN (i.e., the main broad peak in figure 8(a) and (b)). The OASPL directivity is now less within 1.5 dB for all angles, though the levels for $f \geq 10,000$ Hz are still under-predicted at most angles. It turns out that these changes are mostly due to the slight increase in mesh resolution in the near-nozzle region. This was confirmed by a shorter auxiliary simulation on the same 66M cell grid without the bypass flow (not presented). Overall, the LES results suggests that the presence of bypass stream has subtle effects on the noise sources and does altered the radiated sound for the present heated over-expanded supersonic jet. The consistent differences at high frequencies suggest that the fine scale shear layer turbulence near the nozzle is perhaps not duplicated in full with the present computations. More realistic simulation of the core-bypass flow with the center plug and axisymmetric splitter included (rather than modeled with a dual plug-flow inlet profile) could potentially provide more accurate results and further insight on this topic. Nevertheless, these comparisons confirm that the LES captured the relevant flow physics and provide some confidence in the study of the temperature non-uniformity effects presented in the next section.

IV. Preliminary results on the effects of temperature non-uniformity

A. Flow field

Figure 9 show the instantaneous temperature and the velocity magnitude for the afterburning and non-uniform inlet cases. Overall, the simulations exhibit similar flow features than the case at military power conditions without bypass flow presented in the figure 6(a). Independently of the inlet condition, a thin (nominal-turbulent) boundary layer develops in the diverging part of the nozzle and the flow separates from the nozzle internal wall just upstream of the exit to immediately transition to a turbulent shear layer. For the non-uniform case, the cooler flow along the centerline persists significantly downstream of the nozzle before starting to mix out with the hotter outer flow, beginning around 6D from the exit. Again, for all cases, the equal NPR condition results in similar shock structure in the jet plume and same (ideally-expanded) jet Mach number. However, the actual speed of sound, and therefore the jet flow velocity, are different because of the increased temperature. Overall, the visualizations of the cooler/denser and slower central region are reminiscent of the instabilities and mixing of an axisymmetric wake. The radial structure of coherent eddies appear significantly modified and would warrant some further stability analysis.

The contours of the time-averaged and rms streamwise velocity are shown in figure 10, in the midsection plane through a large facet. As expected, the higher jet velocity for the afterburner case leads to a slightly shorter potential core and higher rms levels than for the military power. Aside from these differences, the results for the two cases are similar when non-dimensionalized by $U_j$. The mean contours confirm that the shock structure has remained largely unchanged over the first 5D from the nozzle exit. For $x/D = 5$ to 10, the shock locations are again similar for all the cases but there is an increase in the mean centerline velocity for the non-uniform inlet case, corresponding to the mixing of the outer faster stream. For that simulation, the presence of the cooler, slower flow in the core of the jet seem to reduce the inward spreading of the shear layer, with the outward spreading mostly unaffected, as shown in the bottom plot of figure 10(b). In that figure, it is also clear that the rms levels are reduced compared to the afterburner case, specially in the inner side of shear layer. Additional analysis is ongoing to quantify how the introduction of the inner cooler flow affects the jet development and convective velocity within the shear layer.

B. Radiated noise

Figures 11 shows the noise spectra and directivity on the 45-foot arc from the nozzle exit, for the three main cases without bypass flow. As expected, for the cases with afterburning and non-uniform inlet profiles, the increase in jet velocity drives the increase in noise over a wide range of frequencies and angles. Comparing the military power to afterburner conditions, the peak OASPL increases by 3.9 dB and the radiated power increases by 4.2 dB. There is also a clear shift in the peak noise radiation towards more sideline angles, i.e., from $\phi = 130 - 135^0$ for the military conditions to $115 - 120^0$ for the highly-heated conditions. This shift is consistent with the Mach wave radiation dependence on acoustic Mach number.\textsuperscript{21}

In terms of noise mitigation, the non-uniform temperature case does realize a reduction of 1.6 dB in
Figure 9. Instantaneous temperature $T/T\infty$ (top image) from 1 (black) to 5.5 (white) and velocity magnitude $|u|/c\infty$ (bottom image) from 0.1 (blue) to 3.5 (red).
peak OASPL, along with a 1.1dB reduction in radiated power, as compared to the afterburner case. The drop in OASPL is encouraging, as well as the changes in frequency spectra at the location of peak radiation shown in figure 11(d). The noise reduction from the afterburner to the non-uniform profile is significant over a large frequency range and most inlet angles. The similarities between the non-uniform temperature and afterburner in the high frequency range indicate that the small scale structures are not being effected. The subtle change in slope at the highest range at about 17 kHz (for instance in the figures 11(b), (c) & (d)), corresponds to the expected grid cutoff frequency, assuming a mesh resolution on the FW-H surface of 8 points per wavelength.

C. Spectral proper orthogonal decomposition

It is well known that coherent wavepacket structures within the jet play a central role in noise emissions in supersonic jets, especially in downstream directions. To investigate the impact of temperature variations on these structures and their emitted sound, additional analysis is ongoing for the three jets using a frequency domain version of spectral proper orthogonal decomposition called spectral proper orthogonal decomposition (SPOD). SPOD combines the advantages of the usual spatial version of POD and dynamic mode decomposition, and identifies flow structures that evolve coherently in space and time. To compute SPOD modes from the LES data, we follow the procedure outlined by Towne et al. To simplify the analysis, we assume azimuthal symmetry, which is not strictly true for the faceted nozzle, but we expect this to have negligible impact on the large-scale structures of interest. See Schmidt et al. for a recent application of SPOD to turbulent jets, and an example of the physical insight that can be gained from such an analysis.

The leading SPOD mode for the axisymmetric component of the pressure field at $St = 0.64$ for the military power and afterburner conditions is shown in Figure 12. It is clear that the temperature changes impact both the wavepackets and their associated acoustic radiation. In particular, the change in directivity angle with the increase in flow velocity that was previously observed in figure 11(f) is captured by the SPOD modes, suggesting that this behavior can be related to changes in the properties of the large-scale coherent wavepacket structures. This observation will be further explored in future work using stability analysis of the jet mean flows.
Figure 11. Noise spectra and OASPL directivity for the different operating conditions without bypass flow: military power (---); afterburner (-----); non-uniform inlet (-----)

(a) $\phi = 60^\circ$

(b) $\phi = 90^\circ$

(c) $\phi = 115^\circ$

(d) $\phi = 130^\circ$

(e) $\phi = 150^\circ$

(f) OASPL
To investigate the impact of inlet temperature non-uniformity on the jet flow and noise, large-eddy simulations were performed for heated over-expanded supersonic jets issued from a military-style nozzle, with the unstructured compressible flow solver “Charles” developed at Cascade Technologies. The sharp-throat converging-diverging faceted nozzle is explicitly included in the computational domain, with grid size of order $O(60M)$ cells, using a novel mesh generation paradigm based on the computation of Voronoi diagrams. As part of an initial study done during the Center for Turbulence Research Summer Program 2018 at Stanford University, three operating conditions were simulated: military power, full afterburner, and non-uniform temperature inlet. The latter case corresponds to an annulus of afterburned exhaust and a central stream of military power exhaust.

First, a blind comparison was conducted with the measurements from NASA Glenn Research Center at military power conditions. Overall, the initial LES results were encouraging and compared well with the experiments, exhibiting many of the same prominent flow features, along with reasonable accuracy for the noise predictions. An additional simulation was subsequently performed, with increased resolution in the near-nozzle exit region and a dual plug-flow profile at the inlet to model the experimental core-bypass flow upstream of the nozzle throat. With these changes, the LES flow field and shock structures more closely match the PIV data and the far-field noise predictions are within 1-2 dB of the microphone data for most relevant angles and frequencies. While further improvements in the agreement could potentially be achieved by explicitly including the center plug and axisymmetric splitter in the simulation, these comparisons provided confidence that the main flow physics was appropriately captured in the LES.

With the numerical setup and methodology validated, the afterburning case was then contrasted with the non-uniform inlet temperature case, in an effort to assess potential noise reduction. Here, it is important to note that all the simulations have the same nozzle pressure ratio and there are no changes in nozzle geometry, even for the afterburner conditions. These assumptions lead to the same thrust for all cases and results in similar shock structure in the jet plume. However, for the non-uniform case, the presence of the cooler, slower flow in the core of the jet changes the mixing downstream of the shock train and reduces the inward spreading of the shear layer and its peak fluctuation levels. The radiated noise is also reduced over a large frequency range and for most inlet angles. Overall, a 1.6 dB reduction in peak OASPL and a
1.1 dB reduction in radiated power were realized with the non-uniform inlet conditions, compared to the afterburner conditions. The noise mitigation concept of a non-uniform temperature profile had previously been investigated experimentally only at low temperatures and temperature ratios.\textsuperscript{1–3} The results of the present study indicate that there is merit to the idea when taking into consideration more realistic conditions.

Much work remains to fully analyze the LES data generated by this investigation, along with more concepts worthy of examination to improve the simulations. The assumption of a single ideal gas with constant $\gamma$ used in the present work will be revisited in future computations, since it has been suggested that accounting for changes in specific heat ratio could impact the predicted noise for heated jets.\textsuperscript{27} Modeling of the combustion product properties and mixing with ambient air could potentially improve accuracy and provide further physical insight on the jet behavior in real systems. Finally, there is additional experimental work from Mayo et al.,\textsuperscript{2} where the cooler spot is moved to a wall, creating an asymmetric profile. The new spot location increases shear layer thickness, and a reduction in local peak shear stress levels. The disruption of the shear layer indicates a potential reduction in high-frequency noise. The breadth of follow on work, and the promising preliminary results, combine to augur many exciting ways forward with this work.

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\section*{References}


